CONCETTI "PRELIMINARI"

Giovedì 22/10/15

- PERMUTAZIONI
- DISPOSIZIONI
- COMBINAZIONI
- BINOMIO di NEWTON
- TRIANGOLO di TARTAGLIA (Pascal)

ROMA

$\boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{1} = 4!$

1 2 3  ♡ A B

$(1,2)\quad(1,3)$

$\boxed{7}\ \boxed{6}$

Sottoinsiemi di 3 elementi

$$\frac{7\,6\,5}{3!} = C_{7,3} = \binom{7}{3}$$

$$(a+b)^n$$

$n=2 \quad (a+b)(a+b) = aa + ba + ab + bb$

$n=3 \quad aaa + baa + aba + bba +$
$aab + bab + abb + bbb$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} b^k a^{n-k}$$

pausa di riflessione

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}\frac{(n-k)!}{(n-k)!} =$$

$$= \frac{n!}{k!(n-k)!}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)!(n-\cancel{1}-k+\cancel{1})!} +$$

$$+ \frac{(n-1)!}{k!(n-1-k)!} = (n-1)!\left(\frac{1}{k(k-1)!(n-k-1)!} +\right.$$

$$\left. + \frac{1}{(k-1)!(n-k)(n-k-1)!}\right) =$$

$$= \frac{(n-1)!}{(k-1)!(n-k-1)!}\underbrace{\left(\frac{1}{k} + \frac{1}{n-k}\right)}_{\frac{n}{(n-k)k}} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

—— fine dimostrazione

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} b^k a^{n-k}$$

$$(a+b)^{n+1} = (a+b)\sum_{k=0}^{n} \binom{n}{k} b^k a^{n-k} =$$

$$= \sum_{k=0}^{n} \binom{n}{k} b^k a^{n-k+1} + \underbrace{\sum_{k=0}^{n} \binom{n}{k} b^{k+1} a^{n-k}}_{k+1=h} =$$

$$\sum_{h=1}^{n+1} \binom{n}{h-1} b^h a^{n-(h-1)}$$

$$= \overset{k=0}{a^{n+1}} + \sum_{k=1}^{n} \binom{n}{k} b^k a^{n+1-k} + \overset{h=n+1}{b^{n+1}} + \sum_{h=1}^{n} \binom{n}{h-1} \ldots$$

$$= a^{n+1} + b^{n+1} + \sum_{k=1}^{n} \left(\binom{n}{k} + \binom{n}{k-1} \right) b^k a^{n+1-k}$$

$$= a^{n+1} + b^{n+1} + \sum_{k=1}^{n} \binom{n+1}{k} b^k a^{n+1-k} =$$

$$= \sum_{k=0}^{n+1} \binom{n+1}{k} b^k a^{n+1-k}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 2 3 5 8 13

1

PROBLEMA

Sia $A = \{1, 2, 3, \quad , 11\}$

Quanti sono i sottoinsiemi di A con almeno due elementi che non contengono numeri consecutivi

Contiamo

Coppie

1 $\{1,3\}$ $\{1,4\}$ $\{1,5\}$. $\{1,11\}$ 9

2 $\{2,4\}$ $\{2,5\}$ $\{2,11\}$ 8

3 7 $\binom{10}{2}$

9 $\{9,11\}$ 1

$$\frac{n(n+1)}{2} = \binom{n+1}{2}$$

Terne

$\{1,3,5\}$ $\{1,3,6\}$. $\{1,3,11\}$ 7

$\{1,4,6\}$ 6 $\binom{8}{2}$

$\binom{8}{2}$

$$\binom{10}{2} + \binom{9}{3} + \binom{8}{4} + \binom{7}{5} + \binom{6}{6}$$

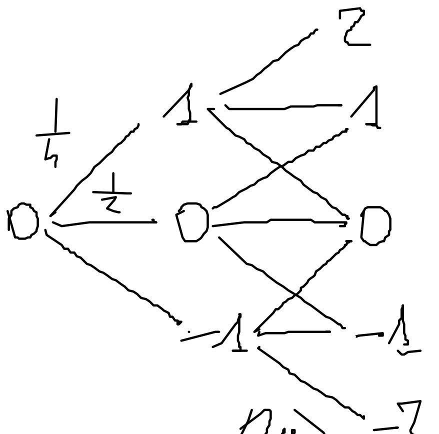
$a, b \longrightarrow a, b-1$

$1, 3 \longrightarrow 1, 2$ $\binom{10}{2}$

$9, 11 \longrightarrow 9, 10$

$a, b, c \longrightarrow a, b-1, c-2$ $\binom{9}{3}$

Data una quantità Q ($Q_0=0$) ad intervalli regolari aumenta di 1 con $p=\frac{1}{4}$, rimane ugu. $p=\frac{1}{2}$, diminuisce di 1 con $p=\frac{1}{4}$. Determina la p. che dopo n passi la quantità sia K



$$p_{22} = \frac{1}{4}\frac{1}{4} = \frac{1}{16}$$

$$p_{21} = \frac{1}{4}\frac{1}{2} + \frac{1}{2}\frac{1}{4} = \frac{4}{16}$$

$$p_{20} = \left(\frac{1}{4}\frac{1}{4}\right)2 + \frac{1}{2}\frac{1}{2} = \frac{6}{16}$$

$$p_{2,-1} = \frac{4}{16}$$

$$p_{2,-2} = \frac{1}{16}$$

$$p_{n,q} \sim \frac{\binom{n}{q}}{4^n} \qquad k = q+n$$

$$p_{nk} = \frac{\binom{2n}{k-n}}{4^n}$$ ⇐ congettura da dimostrare per induzione

Schema di sol. con i polinomi $3 \mid n \qquad n \equiv_3 0$

$$f(x) = \left(\frac{1}{4}x^{-1} + \frac{1}{2}x^0 + \frac{1}{4}x^1\right)^2$$

$$= \frac{1}{16}x^{-2} + \frac{1}{4}x^0 + \frac{1}{16}x^2 + \frac{1}{4}x^{-1} + \frac{1}{8}x^0 + \frac{1}{4}x^1 =$$

$$f(x) = \left(\frac{1}{4}x^{-1} + \frac{1}{2}x^0 + \frac{1}{4}x^1\right)^n =$$

$$= \frac{1}{4^n}\left(\frac{1}{x} + 2 + x\right)^n = \frac{1}{4^n x^n}\left(1+2x+x^2\right)^n =$$

$$= \frac{1}{4^n}\frac{1}{x^n}(1+x)^{2n}$$